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The recent secular variation and the motions at the core surface

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The Earth's magnetic field has been undergoing a remarkably systematic variation during the last 30 years. This variation can be described by a constant time derivative and a step-function second derivative. These parameters are smoothly distributed over the Earth's surface. The step occurred in 1969 and caused the second derivative to change signs for all of the components at most of the magnetic observatories. Similar but less well documented behaviour had been observed around 1900; it seemed to correlate with a jump in the acceleration of the Earth's rotation. We have investigated the motions at the top of the Earth's core that are responsible for the recent magnetic variations by inversion procedures. The motions responsible for the time derivative of the magnetic field can be reasonably well assessed and are dominated by a westward drift term of approximately 0.2°/year, although important poloidal motions are also inferred. The data for the jump in the second derivative are much noisier and the motion accelerations are not as well resolved. The poloidal acceleration terms are still fairly well resolved, but the toroidal motions, especially the zonal motions, are very poorly resolved. No firm conclusion about an acceleration of the westward drift can be given on the basis of this analysis. The inversions give evidence that the motions for the lower modes are a strongly decreasing function of their order.

1. Introduction

It has been shown (Courtillot *et al.* 1978, 1979; Ducruix *et al.* 1980; Malin *et al.* 1982) that the secular variation of the geomagnetic field during the timespan 1950–80 could be approximated, in most observatories, by very simple curves. Over this time interval large changes in the values of the secular variation (s.v.) field have occurred (by a factor of two in some places). For such timescales, Roberts & Scott (1965) argue that the core advects the geomagnetic field as a perfect conductor. This hypothesis was first adopted by Kahle *et al.* (1967*a*, *b*) who tried to determine the velocity of the fluid at the core surface. Then Backus (1968) examined the question of the existence and uniqueness of a motion able to generate the given variation field from a given main field, and Booker (1969) applied Backus's theory. More recently, Muth & Benton (1981) and Whaler (1980) have again addressed the problem of the determination of the velocity field.

In the present paper we shall see, from the same hypothesis (the Hide–Roberts–Scott hypothesis), but with a special attention paid to inversion techniques, what can be inferred about the motions of the fluid near the surface of the core; we shall consider separately the velocity field and the acceleration field, as is suggested by the particularly simple features displayed by the secular variation of the geomagnetic field for the period under consideration.

2. The description of the recent secular variation

It has been shown that the secular variation of the geomagnetic field exhibited a jump at about 1969 all over the world (Courtillot *et al.* 1978; Ha Duyen *et al.* 1981; Le Mouël *et al.* 1982; Malin *et al.* 1982): the three second time derivatives $\ddot{X}(t)$, $\ddot{Y}(t)$, $\ddot{Z}(t)$ of the north, east and vertical components suddenly change their values at this epoch. We have completed this analysis over the 1950–1980 timespan, using all the available observatory data provided by the World Data Center A (Boulder, Colorado). We shall only give the general conclusions of this analysis in this paper.

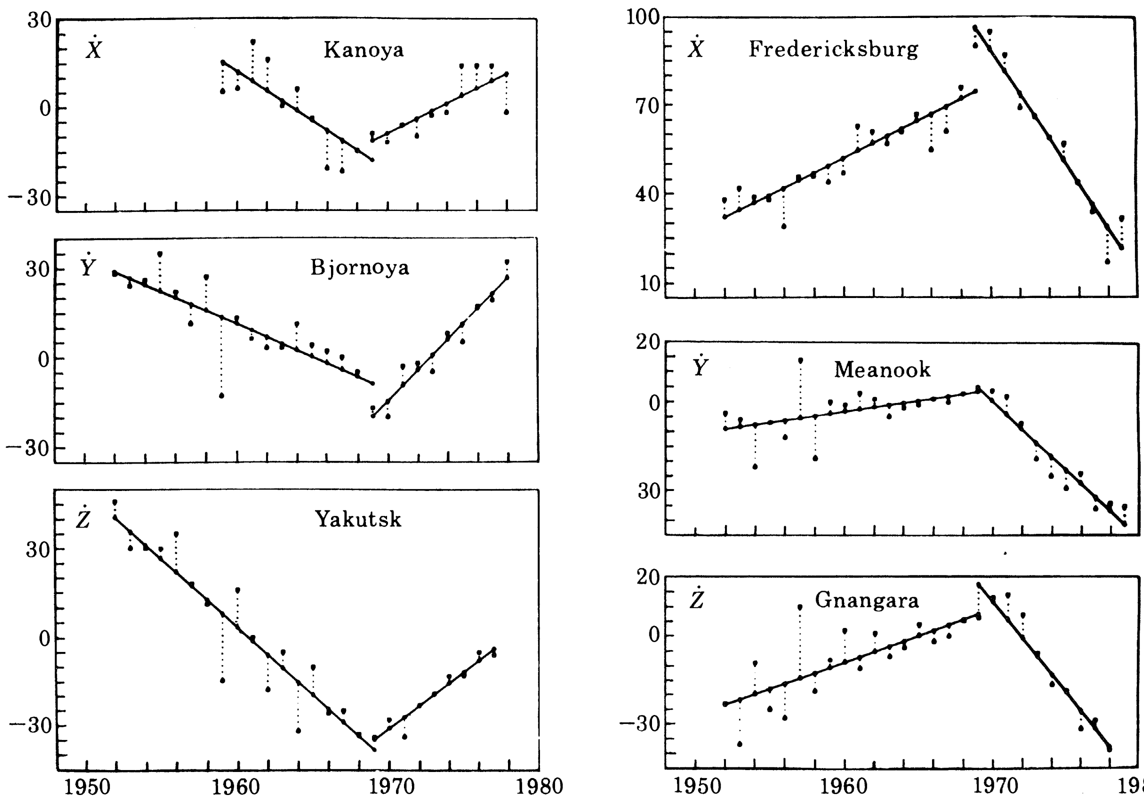

Figure 1. Examples of representation of the secular variation $\dot{X}$, $\dot{Y}$, $\dot{Z}$ by two segments of straight line in some observatories. Points are values of $\dot{E}(n) = E(n) - E(n-1)$ ($E = X, Y, Z$; n is the number of the year).

(*a*) In most observatories the s.v. data, $\dot{X}(t)$, $\dot{Y}(t)$, $\dot{Z}(t)$ may be represented, as a first (but in many places very good) approximation, by two segments of straight line with the apex in 1969. Equivalently the main field components themselves, $X(t)$, $Y(t)$, $Z(t)$, may be represented by two parabolas joining in 1969. Figure 1 illustrates this observation in some observatories. Let $\ddot{X}^-$, $\ddot{Y}^-$, $\ddot{Z}^-$ be the slopes (in nanoteslas/year²) of the straight lines before 1969, respectively for the $\dot{X}$, $\dot{Y}$, $\dot{Z}$ components, and $\ddot{X}^+$, $\ddot{Y}^+$, $\ddot{Z}^+$ the slopes after 1969; let $\boldsymbol{\Gamma}^-(\ddot{X}^-, \ddot{Y}^-, \ddot{Z}^-)$ and $\boldsymbol{\Gamma}^+(\ddot{X}^+, \ddot{Y}^+, \ddot{Z}^+)$ be the acceleration vectors, constant at each point in our approximation. Within this approximation, the secular variation field $\dot{\boldsymbol{B}}(P, t)$ may be written in the form:

$$\dot{\boldsymbol{B}}(P,t) = \begin{cases} \dot{\boldsymbol{B}}(P,t_0) + \boldsymbol{\Gamma}^-(P)\,(t-t_0), & t < t_1; \\ \dot{\boldsymbol{B}}(P,t_0) + \boldsymbol{\Gamma}^-(P)\,(t_1-t_0) + \boldsymbol{\Gamma}^+(P)\,(t-t_1), & t > t_1; \end{cases} \tag{1}$$

t_0 being 1950 and t_1 1969.

(*b*) The geometry of the acceleration field, as illustrated by figure 2, appears to have a rather simple worldwide character. When comparing the $\ddot{X}^+$, $\ddot{Y}^+$, $\ddot{Z}^+$ with the $\ddot{X}^-$, $\ddot{Y}^-$, $\ddot{Z}^-$ maps (not represented here), it appears that the geometry of the acceleration field tends to be fixed, although it undergoes a sign reversal. Some similarities also exist between the s.v. and the acceleration geometries. Let us recall that a similar event, although perhaps less sharp, occurred at the beginning of the century (Courtillot *et al.* 1978; Le Mouël *et al.* 1981).

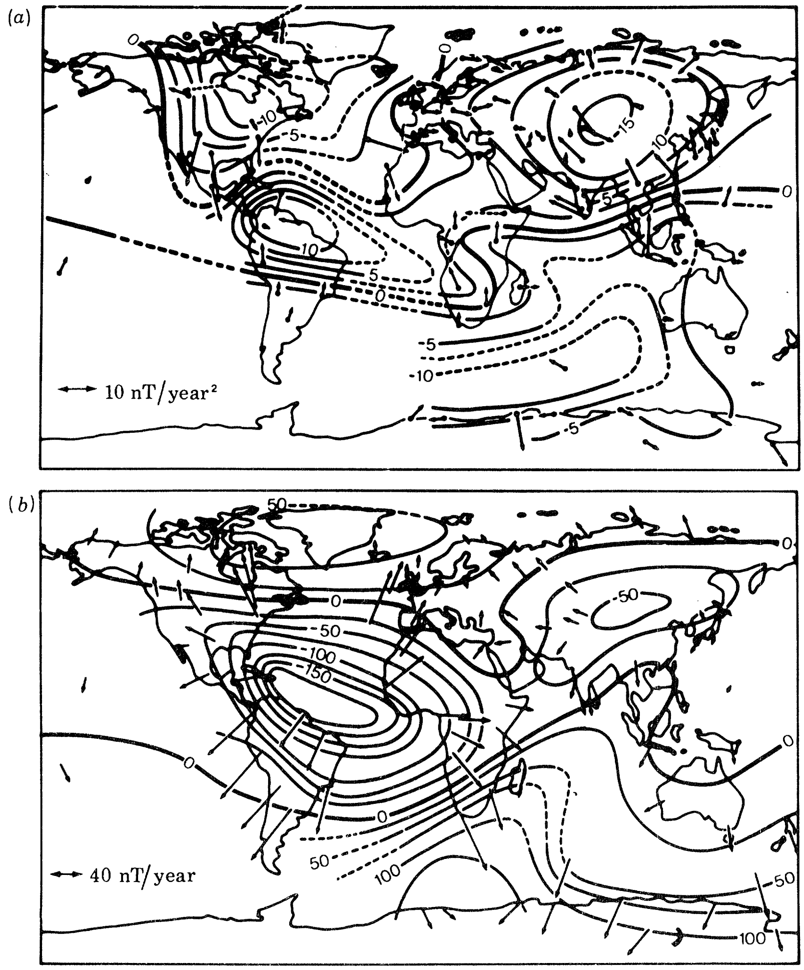

FIGURE 2. Sketch of the geometry of (*a*) the acceleration field, (*b*) the secular variation field. Contours are isovalue lines of $(\ddot{Z}^+ - \ddot{Z}^-)$ in nanoteslas per year squared (*a*) and $\dot{Z}$ in nanoteslas per year (*b*). Arrows represent the horizontal vectors $(\ddot{\boldsymbol{H}}^+ - \ddot{\boldsymbol{H}}^-)$ (*a*) or $\dot{\boldsymbol{H}}$ (*b*).

3. Hypothesis of advection of lines of force

For time constants on the order of 10 years, the material of the core ($r < b$) may be assumed to be a perfect conductor. Then the temporal variations of the geomagnetic field outside the core only depend on the motion $\boldsymbol{u}$ at its surface, as first demonstrated by Bondi & Gold (1950) (in fact at the surface of the main stream, just below a boundary layer through which $\boldsymbol{B}$ is unchanged (Roberts & Scott 1965)). In this approximation the secular variation field and the main field are related by the equation (in the main stream)

$$\dot{\boldsymbol{B}} = \partial \boldsymbol{B}/\partial t = \mathrm{rot}\,(\boldsymbol{u} \wedge \boldsymbol{B}). \tag{2}$$

With the same approximation the electric field $\boldsymbol{E}$ may be written:

$$\boldsymbol{E} = -(\boldsymbol{u} \wedge \boldsymbol{B}). \tag{3}$$

And the observed poloidal field $\partial \boldsymbol{B}/\partial t$ only depends on the toroidal part $\boldsymbol{E}_t$ of $\boldsymbol{E}$.

Owing to the linearity of (2) and to the fact that over a timespan Δt of 30 years, $|\boldsymbol{B}\Delta t| \ll |\boldsymbol{B}|$, we may expect that the velocity field $\boldsymbol{u}$ at the surface of the core has the form (despite the non-uniqueness of the motion $\boldsymbol{u}$; see §5):

$$\boldsymbol{u}(P, t) = \begin{cases} \boldsymbol{u}(P, t_0) + \gamma^-(P)\ (t-t_0), & t < t_1; \\ \boldsymbol{u}(P, t_0) + \gamma^-(P)\ (t_1-t_0) + \gamma^+(P)\ (t-t_1), & t > t_1. \end{cases} \tag{4}$$

This might appear to be trivial. But it should be emphasized, first, that γ suddenly reverses in sign in most places at the core surface at the same epoch, and, second, that the quantities $\gamma^-(t-t_0)$ and $\gamma^+(t-t_1)$ may be of the same order of magnitude as $\boldsymbol{u}$ itself.

We have performed an inversion of the s.v. data $\dot{\boldsymbol{B}}$ and of the jump data $(\boldsymbol{\Gamma}^+ - \boldsymbol{\Gamma}^-)$ to determine the configurations of the $\boldsymbol{u}$ field and of the acceleration jump field $(\gamma^+ - \gamma^-)$.

The horizontal velocity field $\boldsymbol{u}$ at the core surface may be written in the form of the sum of a poloidal part $\boldsymbol{s}$ and a toroidal part $\boldsymbol{t}$ (see, for example, Kahle *et al.* 1967*a*; Backus 1968):

$$\boldsymbol{u} = \boldsymbol{s} + \boldsymbol{t} = \nabla S + \boldsymbol{n} \wedge \nabla T; \tag{5}$$

∇ is the horizontal gradient operator $(\theta, \partial/\partial\theta, (1/\sin\theta)\ (\partial/\partial\phi))$, S and T two scalar functions of colatitude θ and longitude ϕ, $\boldsymbol{n}$ the unit radial vector $\boldsymbol{r}/r$. Vectors $\boldsymbol{s}$ and $\boldsymbol{t}$ are also called respectively irrotational and solenoidal.

As usual, we shall expand S and T in superficial harmonic functions

$$Y_p(Y_p(\theta, \phi) = Y_n^m(\theta, \phi) = P_n^{|m|}(\cos\theta)\ e^{im\phi}).$$

Then

$$\boldsymbol{u} = \sum_p s_p \boldsymbol{\sigma}_p + \sum_p t_p \boldsymbol{\theta}_p \tag{6}$$

with

$$\boldsymbol{\sigma}_p = \nabla Y_p, \quad \boldsymbol{\theta}_p = \boldsymbol{n} \wedge \nabla Y_p. \tag{7}$$

The coefficients s_p and t_p of the elementary vectors (or modes) $\boldsymbol{\sigma}_p$ and $\boldsymbol{\theta}_p$ are in metres per second. We will also give them in degrees per year (at the core surface 1° per year = 2 mm s^{-1}).

4. A priori considerations about the velocity field $\boldsymbol{u}$

Before presenting the inversion and its results, we shall turn briefly to some *a priori* considerations about the $\boldsymbol{u}$ motion, which will guide us to some extent in the later computations.

(*a*) In previous papers (Le Mouël & Courtillot 1981; Le Mouël *et al.* (1981) we have considered a simple model of Earth's core (a Bullard's model): a thin external shell (thickness h) overlies an inner core; exchanges of fluid and then of angular momentum are allowed between the two. We were led to suppose that h is much less than the radius, b, of the core (in fact of the order of a few hundreds of kilometres). In such an approximation, the velocity field $\boldsymbol{u}$ of (6) is the velocity of the fluid in the thin outer layer which is fed from below because $\boldsymbol{u}$ has an irrotational ingredient. The net angular momentum of the outer shell with respect to the rotation axis Oz (unit vector $\boldsymbol{z}$) of the Earth is then

$$\Sigma_z = \int_{\text{outer layer}} (\boldsymbol{r} \wedge \boldsymbol{u})\ \rho\, \mathrm{d}v \cdot \boldsymbol{z} = -\tfrac{8}{3}\pi h \rho b^3 t_1^0. \tag{8}$$

Only the body rotation $\boldsymbol{\theta}_1^0$ contributes to the net momentum (ρ is the density of the core).

Let $\boldsymbol{\Omega}$ be the angular bodily rotation of the Earth ($\Omega \approx 7.3 \times 10^{-5}\ \mathrm{s^{-1}}$). The rate of angular momentum transfer from the inner to the outer core is

$$\frac{\mathrm{d}\boldsymbol{\Sigma}}{\mathrm{d}t} = \int_{r=b-h} \boldsymbol{r} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{r})\, \rho u_\mathrm{r} \mathrm{d}S,$$

u_r being the radial component of the flow in the inner core just below $r = b-h$. It comes out as (always in our approximation):

$$\mathrm{d}\Sigma_z/\mathrm{d}t = \tfrac{16}{5}\pi\rho h b^3 \Omega s_2^0. \tag{9}$$

This quantity is also the resulting torque with respect to the Oz rotation axis of the Coriolis forces $-2\rho(\boldsymbol{\Omega} \wedge \boldsymbol{u})\, \mathrm{d}v$ acting on the external layers. It should be noted that only the $\boldsymbol{\sigma}_2^0$ elementary motion (the poloidal P_2^0 mode) contributes to the angular momentum exchange.

As it is generally accepted that part of the secular variation is due to a general westward drift of the geomagnetic field, and as it is legitimate to associate such a drift of the field with a bodily rotation of the outer layers of the core, we will of course expect a $\boldsymbol{\theta}_1^0$ mode in $\boldsymbol{u}$. As the drift rate appears to be rapidly varying (Harwood & Malin 1976; Le Mouël *et al.* 1981 (see, however, the conclusion of this paper)), there is a transfer of angular momentum between outer and inner layers and we expect a $\boldsymbol{\sigma}_2^0$ mode in $\boldsymbol{u}$. In the frame of the elementary model we considered (Le Mouël & Courtillot 1982) it is possible to estimate the ratio s_2^0/t_1^0. It comes out that this ratio does not depend on h. With the numerical values of the model we obtained $s_2^0 \approx 3 \times 10^{-2}\, t_1^0$.

(*b*) When looking at a map of the secular variation field, the $\dot{X}$ isolines appear to be (roughly) antisymmetrical with respect to the equator, while the $\dot{Z}$ isolines are symmetrical. The reverse is true for X and Z. This observation is a hint of a north–south meridian motion. A $\boldsymbol{\sigma}_1^0$ motion acting on a centred axial dipole field (P_1^0) generates a P_2^0 poloidal field. Indeed, the geometry of the $\boldsymbol{B}$ field, in its broad lines, reveals such a P_2^0 component and $\boldsymbol{u}$ is expected to contain some $\boldsymbol{\sigma}_1^0$ mode.

(*c*) More generally, the worldwide pattern of the acceleration field $\ddot{\boldsymbol{B}}$ and the relative simplicity of its geometry lead us to expect that the motion $\boldsymbol{u}$ itself has a worldwide organization and that (6) will be dominated by low-order modes.

5. Computation of the $\boldsymbol{u}$ velocity

The method we use is similar in its principle to that of Kahle *et al.* (1967*a*, *b*) in their pioneer work. But here the analysis has been pushed a step further and advantage has been taken of improvements in the data and of advances in theoretical work made after the papers by Kahle *et al.*

First we continue the radial component B_r of the main field from the Earth surface ($r = a$) down to the core surface ($r = b$) by making the assumption that the mantle is insulating:

$$(B_\mathrm{r})_{r=b} = \sum_i C_l \left(\frac{a}{b}\right)^{n+1} Y_l(\theta, \phi), \tag{10}$$

the coefficients C_l (l being the doublet (n, m)) being the coefficients of a recent model (based on Magsat data, and limited to degree and order 10).

Let us now expand the toroidal electric field in elementary toroidal vectors (see (6) and (7)):

$$\boldsymbol{E}_\mathrm{t} = \sum_i e_i \boldsymbol{\theta}_i. \tag{11}$$

The coefficients e_i of (11) have for expressions

$$b^2N^2(\boldsymbol{\theta}_i)\, e_i = -\int_{r=b} (\boldsymbol{u} \wedge \boldsymbol{B}) \cdot \boldsymbol{\theta}_i \, dS = -\int_{r=b} (\boldsymbol{\theta}_i^* \wedge \boldsymbol{u})_r\, B_r \, dS, \tag{12}$$

the r subscript denoting the radial component, * the imaginary conjugate and $N^2(\boldsymbol{\theta}_i)$ being the norm of the $\boldsymbol{\theta}_i$ vector.

Considering separately the contributions of the poloidal and the toroidal ingredients of $\boldsymbol{u}$ (equation (6)), it comes out as

with

$$\left.\begin{aligned} e_i &= f_i + h_i, \\ f_i &= \sum_{p,\,l} f_i(p, l), \\ h_i &= \sum_{p,\,l} h_i(p, l) \end{aligned}\right\} \tag{13}$$

and

$$N^2(\boldsymbol{\theta}_i) f_i(p, l) = s_p C_l \left(\frac{a}{b}\right)^{n+1} \int_{r=1} \left(\frac{1}{\sin^2\theta}\frac{\partial Y_i^*}{\partial\phi}\frac{\partial Y_p}{\partial\phi} + \frac{\partial Y_i^*}{\partial\theta}\frac{\partial Y_p}{\partial\theta}\right) Y_l \, dS, \tag{14}$$

$$N^2(\boldsymbol{\theta}_i)\, h_i(p, l) = t_p C_l \left(\frac{a}{b}\right)^{n+1} \int_{r=1} \frac{1}{\sin\theta}\left(\frac{\partial Y_i^*}{\partial\phi}\frac{\partial Y_p}{\partial\theta} - \frac{\partial Y_i^*}{\partial\theta}\frac{\partial Y_p}{\partial\phi}\right) Y_l \, dS. \tag{15}$$

Integrals similar to those of (14) and (15) were first derived by Gaunt (1929), then by Elsasser (1946) and Bullard & Gellman (1954). They have been more recently reviewed by Scott (1969). They have been reduced to the standard Gaunt and Elsasser integrals. We have evaluated them numerically and checked the results by comparing them with Scott's values. It is known that many of them are null, depending on the values of the three doublets of indices i, p, l. We shall not detail here these selection rules (see Scott 1969).

The observed field is the secular variation field, $\dot{\boldsymbol{B}}$, or the secular acceleration field, $\ddot{\boldsymbol{B}}$, at the surface of the Earth (for $\ddot{\boldsymbol{B}}$ we have also considered as data a model of secular variation).

The expression of $\dot{\boldsymbol{B}}$ is derived in a straightforward fashion from the expression of $\boldsymbol{E}_t$ (formulae (2) and (3)):

$$\dot{\boldsymbol{B}} = -\sum_i n e_i \nabla \left\{\left(\frac{b}{r}\right)^{n+1} Y_i(\theta, \phi)\right\}, \tag{16}$$

n being the degree of the doublet of indices $i(n, m)$.

All these equations can now be reduced to a single one. Let $\dot{\mathscr{B}}$ be the vector formed with the observed values of the secular variation $\dot{X}(S_i)$, $\dot{Y}(S_i)$, $\dot{Z}(S_i)$ in the K observatories S_i ($K \approx 140$), $\mathscr{U}$ the vector formed with the coefficients s_p and t_p of the velocity field (formula (6)). We can write

$$\mathscr{D}\mathscr{U} = \dot{\mathscr{B}}, \tag{17}$$

$\mathscr{D}$ being some matrix, the coefficients of which are derived from (16), (15), (14), (13).

Non-uniqueness

Let us first recall some results about the non-uniqueness of the determination of the velocity field $\boldsymbol{u}$ from the observed secular variation field (in the Roberts & Scott approximation). At the same time we shall extend slightly the previous analyses. Besides a compatibility relation between $\dot{\boldsymbol{B}}$ and $\boldsymbol{B}$, Backus (1968) has established that the vectors of the null-space U_0 (containing the motions $\boldsymbol{u}$, which create no variation field outside the core) are of the form

$$\boldsymbol{u}_0 = \boldsymbol{n} \wedge \nabla\psi / B_r, \tag{18}$$

ψ being any sufficiently regular function of θ and ϕ. So $B_r \boldsymbol{u}_0$ is toroidal. We have examined if the vector $\boldsymbol{u}_0$ itself could be toroidal or poloidal and concluded the following (Madden & Le Mouël 1981).

(*a*) The toroidal vectors $\boldsymbol{u}_{0t}$ of U_0 are necessarily of the form

$$\boldsymbol{u}_{0t} = \boldsymbol{n} \wedge \nabla F(B_r),$$

F being any regular function. All of them have the same lines of force on the sphere ($r = b$) and these lines of force are the B_r isolines. We shall return later to some consequences of this particular geometry.

(*b*) All the poloidal members $\boldsymbol{u}_{0p}$ of U_0 are pathological, i.e. they have infinities on the equator line(s) $B_r = 0$.

Inversion

We first weighted the data $\dot{X}(S_i)$, $\dot{Y}(S_i)$, $\dot{Z}(S_i)$ (or $\ddot{X}(S_i)$, $\ddot{Y}(S_i)$, $\ddot{Z}(S_i)$). Several weightings were tried successively. In a general way the weight given to a given component at a given station S_i is inversely proportional to the square of the noise ν^2 for this component and this station and proportional to the distance R from station S_i to the next closest one:

$$p_i \approx (\nu^2/R)^{-1},$$

and the noise ν^2 is defined as the mean square value of the difference between the $X(t)$ series (and likewise for Y, Z) and a double parabola representation (see §2). Let R_{BB} be the diagonal matrix of the p_i^{-1}.

When trying to invert (17) it appears that some damping is necessary: we have introduced an '*a priori* $\boldsymbol{u}$ variance matrix', R_{UU}, to damp out higher modes in (6) more than lower modes (this procedure is in agreement with our *a priori* views about the prominence of lower order terms, as exposed at the end of §4). For example, R_{UU}^{-1} has been chosen as the diagonal matrix formed with elements $dn(n+1)$, d being a numerical factor characterizing the global strength of the damping and n being the degree of the elementary mode $\boldsymbol{\sigma}_p$ or $\boldsymbol{\theta}_p$. Furthermore in some rare cases we have chosen an *a priori* motion $\boldsymbol{u}$, say $\boldsymbol{u}_1$, and tried to get a solution 'not too far from $\boldsymbol{u}_1$'. In fact, each time this procedure was used, $\boldsymbol{u}_1$ was chosen as the 'drift term' $\boldsymbol{\theta}_1^0$.

So we have minimized:

$$(\dot{\mathscr{B}} - \mathscr{D}\mathscr{U})^T R_{BB}^{-1}(\dot{\mathscr{B}} - \mathscr{D}\mathscr{U}) + (\mathscr{U} - \mathscr{U}_1)^T R_{UU}^{-1}(\mathscr{U} - \mathscr{U}_1), \tag{19}$$

U_1 being the vector formed with the coefficients s, t of $\boldsymbol{u}_1$. We obtain the maximum likelihood solution

$$\mathscr{U} = (\mathscr{D}^T R_{BB}^{-1} \mathscr{D} + R_{UU}^{-1})^{-1} \mathscr{D}^T R_{BB}^{-1}(\dot{\mathscr{B}} - \mathscr{D}\mathscr{U}_1) + \mathscr{U}_1. \tag{20}$$

The resolution matrix and the noise on the coefficients s and t of $\boldsymbol{u}$ have been computed by usual formulae.

Because of the term $\mathscr{U}^T R_{UU}^{-1} \mathscr{U}$ in (19) the solution $\boldsymbol{u}$ tends to be a minimum norm solution. We have seen that the poloidal members of the null-space U_0 are pathological. Now our solution $\boldsymbol{u}$ is regular; it is to be expected that the poloidal part of u will be well determined. On the contrary, the regular velocity field $\boldsymbol{t}$ may contain some contribution from U_0. But, as (20) is a minimum $|\boldsymbol{u}|$ solution, it may be hoped that this contribution will be reduced. Nevertheless we will soon verify that the poloidal terms are indeed better resolved than the toroidal ones.

The expansion for B_r is limited to degree and order 10. For $\boldsymbol{u}$, the degree has been limited to degree 5 (we shall conclude that the amplitude of the modes falls off with their degree). For $\dot{\boldsymbol{B}}$, we have limited the expansion to degree 8, after checking that it was sufficient.

Many inversions with the use of different data sets, different station weightings R_{BB}^{-1}, different *a priori* R_{UU} matrices have been performed. We shall only give one example of inversion for each case (s.v. data and jump data) and comment briefly on the general laws that may be deduced from the whole set of inversions.

S.v. data

(*a*) The map in figure 3 represents the $\boldsymbol{u}$ field obtained as indicated above from the values of the secular variation field $\dot{X}$, $\dot{Y}$, $\dot{Z}$ observed at about 140 observatories in 1973. Other sets of data have been inverted: secular variation data in the same observatories for 1969 and models of s.v. for the time interval 1965–80 (the data are then the coefficients C_l of the s.v. model which is limited to degree 8). The general conclusions are as follows.

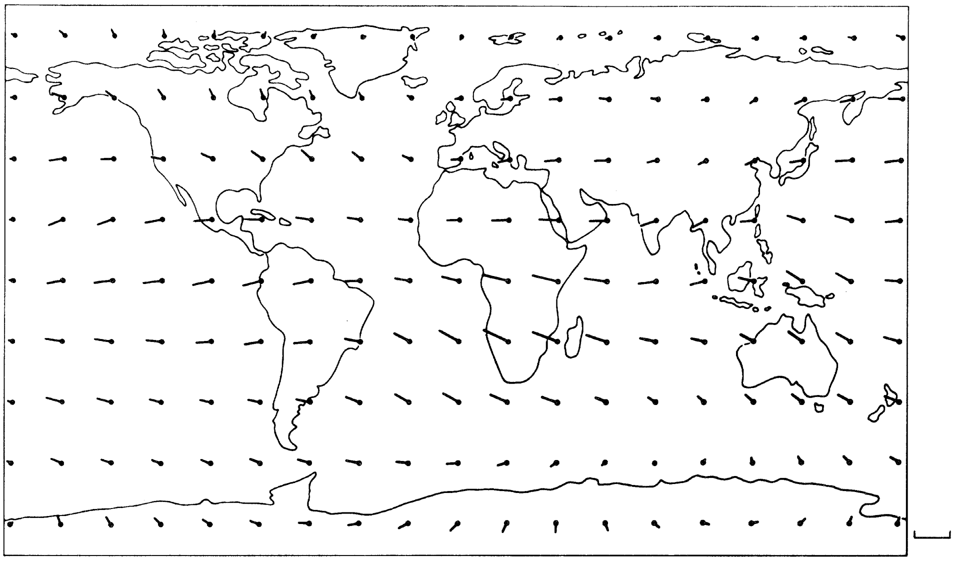

FIGURE 3. Velocity field, $\boldsymbol{u}$, at the surface of the core, 1973 secular variation data. The arrows represent $\boldsymbol{u}$ at the nodes of a 20° × 20° grid. The solid bar represents 0.5°/year.

(i) The predicted s.v. field $\dot{\boldsymbol{B}}_p(S_i)$ (given by $\dot{\mathscr{B}} = \mathscr{D}\mathscr{U}$) fits the observed s.v. field $\dot{\boldsymbol{B}}(S_i)$ with an r.m.s. error of only 6 nT/year (17 % of the r.m.s. value of $\dot{\boldsymbol{B}}(S_i)$).

(ii) Despite the fact that the different data sets used are independent, the corresponding estimates of $\boldsymbol{u}$ are in good agreement. The coefficients t_1^0 (the coefficient of the westward drift that appears clearly on figure 3), s_1^0 (the south–north motion, see §4), s_2^0 (responsible for the angular momentum transfer, see §4) are respectively of the order of 0.20, 0.03 and 0.02°/year. The signs of s_2^0 and t_1^0 are in agreement with the arguments of §4.

(*b*) As a general rule, the amplitude of the different modes falls off with the degree; different tests lead us to believe that this is a significant result (and not an artefact due to damping) and that the motion at the surface of the core is basically of low order. Of course this conclusion cannot be definitively established, in particular because the small-scale components of the magnetic field at the surface of the core cannot be derived from the observations at the Earth's surface.

(i) Poloidal terms are better resolved than toroidal ones: 23 poloidal eigenvectors but only 10 toroidal eigenvectors out of 35 are used in the fit $\dot{\boldsymbol{B}}_p$. They are also less noisy (for instance, for poloidal terms of degree $\leqslant 4$ the estimated noise on a coefficient is one order of magnitude less than the coefficient itself). This difference is a consequence of the nature of the null-space U_0. In particular, it is obvious that the drift term $t_1^0\,\boldsymbol{\theta}_1^0$ has a non-null projection in U_0, so this important term is theoretically among the less well resolved ones (although we observe a fair agreement between our different estimates).

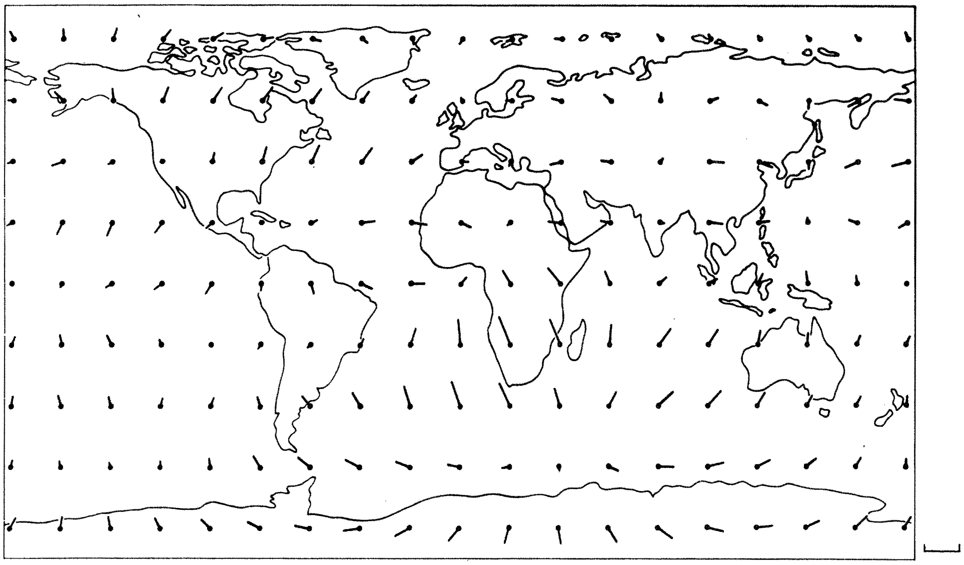

FIGURE 4. Poloidal part, $\boldsymbol{s}$, of the velocity field, $\boldsymbol{u}$, of figure 3. The solid bar represents 0.2°/year.

(ii) When comparing our results with those of Kahle *et al.*, one observes at the same time some striking resemblances such as the big upwelling focus south of Africa which is the most noticeable feature of the map of the poloidal part $\boldsymbol{s}$ of $\boldsymbol{u}$, represented on figure 4, and some large differences. In particular the value that Kahle *et al.* obtained for the drift term is plainly smaller than ours.

Jump data

We have inverted in the same way the values $(\ddot{X}^+ - \ddot{X}^-)$, $(\ddot{Y}^+ - \ddot{Y}^-)$, $(\ddot{Z}^+ - \ddot{Z}^-)$ measured in the same observatories, i.e. the acceleration jump data. Many attempts have been made by using different weighting matrices R_{BB}^{-1} and motion variance matrices R_{UU}. The results are as follows.

(*a*) The coefficients s, t of $(\gamma^+ - \gamma^-)$ (in metres per second squared or degrees per year squared) are much noisier than the ones of $\boldsymbol{u}$ and the higher-order estimates are clearly worthless. The r.m.s. error of the fits $(\ddot{\boldsymbol{B}}_p^+(S_i) - \ddot{\boldsymbol{B}}_p^-(S_i))$ is more than half the r.m.s. value of the observed jump field.

(*b*) The poloidal acceleration terms are still fairly well resolved but the toroidal terms, especially the zonal ones and among them the drift term t_1^0, are very poorly resolved (18 poloidal eigenvectors out of 35 but only 7 toroidal eigenvectors out of 35 were used in the fits). The poloidal acceleration fields derived from different inversions are in fact in reasonable agreement; one of them is illustrated in figure 5. But, contrary to our expectation, our analysis of the jump data does not provide us with any firm conclusion about the drift acceleration related to the jump.

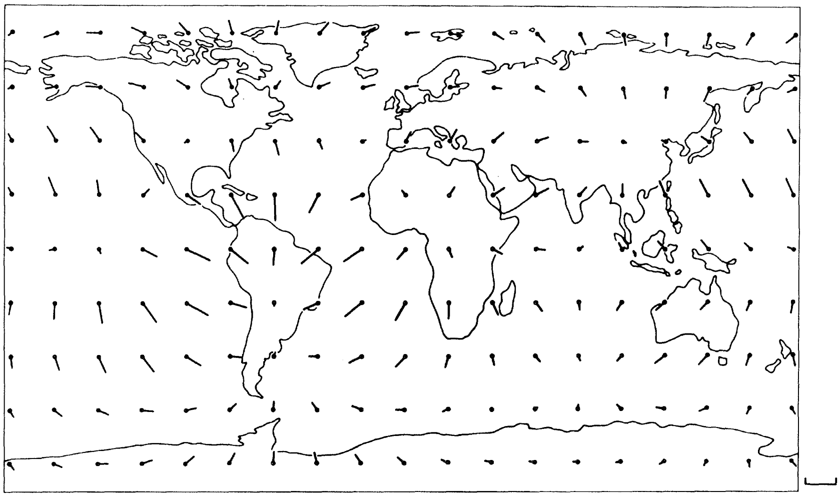

FIGURE 5. Poloidal part of the acceleration jump field, $(\gamma^+ - \gamma^-)$. The solid bar represents 0.025°/year.

6. Conclusion

Our original intent in this study was to provide estimates of the westward drift acceleration from the magnetic secular variation data that could be used to predict changes in the Earth's rate of rotation. Problems arising from the non-uniqueness of the inversion and noise in the data have frustrated this endeavour. The study has, however, led to several positive conclusions.

1. Poloidal motions at the top of the core can be definitely identified. This does not necessarily prevent the top core layer from being convectively stable, as we may be observing penetrative convection.

2. The low-order modes dominate the motion for the scale of motions studied (fifth order) and the westward drift term is an order of magnitude larger than any other harmonic.

3. The 1969 acceleration jump is truly universal, even though the details of the motion accelerations elude us.

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